# SCALING CUBES

## A Sense of Scale

[30 minutes]

Most of us have seen a horror movie or a cartoon featuring a giant insect or spider, maybe one big enough to eat a human being. Why don't insects this large really exist? The answer has to do with the rules of scale. An insect takes in oxygen through the exoskeleton that covers its body. A giant insect would need more oxygen than its surface area could absorb. In other words, it would not have enough surface area for its volume.

One of the basic ideas about scale is how changing the length, width, and height of a three-dimensional object affects its surface area and its volume. In this exercise, students will build bigger and bigger cubes to understand these scaling relationships.

**Materials:**

- Cubes (centimeter cubes, Multilinks, or sugar cubes)
- Paper
- Pencil
- Balance or scale (optional)

**Try This:**

Step 1 Observe one cube. Let the length of its side equal one unit (s=1). How many faces does the cube have? Given that the length of the side is one, what is the surface area of the cube? Recall that the area of a rectangle is length times width, or for a square, side times side ($s^2$).

Step 2 What is the volume of this cube? Recall that volume is length times width times height, or for a cube, side times side times side ($s^3$).

Step 3 Design a table for keeping track of your cube data, with columns for "side length," "surface area," and "volume." If you have access to a balance or scale, add a column for "mass" to your table. Enter the numbers you got for the starter cube with a side length of one unit.

| | Side Length | Surface Area | Volume | Mass |
|---|---|---|---|---|
| Starter Cube | | | | |
| Double Cube | | | | |
| Triple Cube | | | | |

Step 4 Build a cube whose side is equal to two units. Let's call this a "double cube." How many cubes did you use? Find the surface area and the volume of this cube. If you have a scale, find the mass of the double cube. Enter these numbers into your table.

Step 5 Build a cube whose side is equal to three units. Let's call this a "triple cube." How many cubes did you use? Find the surface area and the volume of this cube. If you have a scale, find the mass of the triple cube. Enter these numbers into your table.

Step 6 You have increased the side length by one each time, s = 1, 2, 3. Do you see a pattern in how the surface area changes? Do you see a pattern in how the volume changes? If you have numbers for mass, what pattern do those numbers follow? Can you predict what the "quadruple cube" (s = 4) results will be?

Step 7 If you have enough cubes, work with a partner to build the "quadruple cube" to test your hypothesis.

Notice that as you build the larger cubes, they completely fill the space leaving no gaps between any of the smaller cubes. Cubes are three-dimensional shapes that *tessellate* three-dimensional space—completely filling it leaving no gaps. Can you think of other shapes that would tessellate three-dimensional space?

## What's Going On?

As we increase the length of the side of the cube, the surface area and the volume also increase. The formula for finding the surface area of the cube is $6s^2$. We multiply the number of faces (6) times the area of each face ($s^2$). The formula for finding the volume of the cube is $s^3$. The volume of a larger cube (double cube, triple cube, etc.) is also equal to the volume of one small cube times the number of small cubes you used to make it.

*From the National Council of Teachers of Mathematics (NCTM)*

## SCALING CUBES

Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships:

- Precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties;
- Understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects.

Apply appropriate techniques, tools, and formulas to determine measurements:

- Select and apply techniques and tools to accurately find length, area, volume, and angle measures to appropriate levels of precision;
- Solve problems involving scale factors, using ratio and proportion.