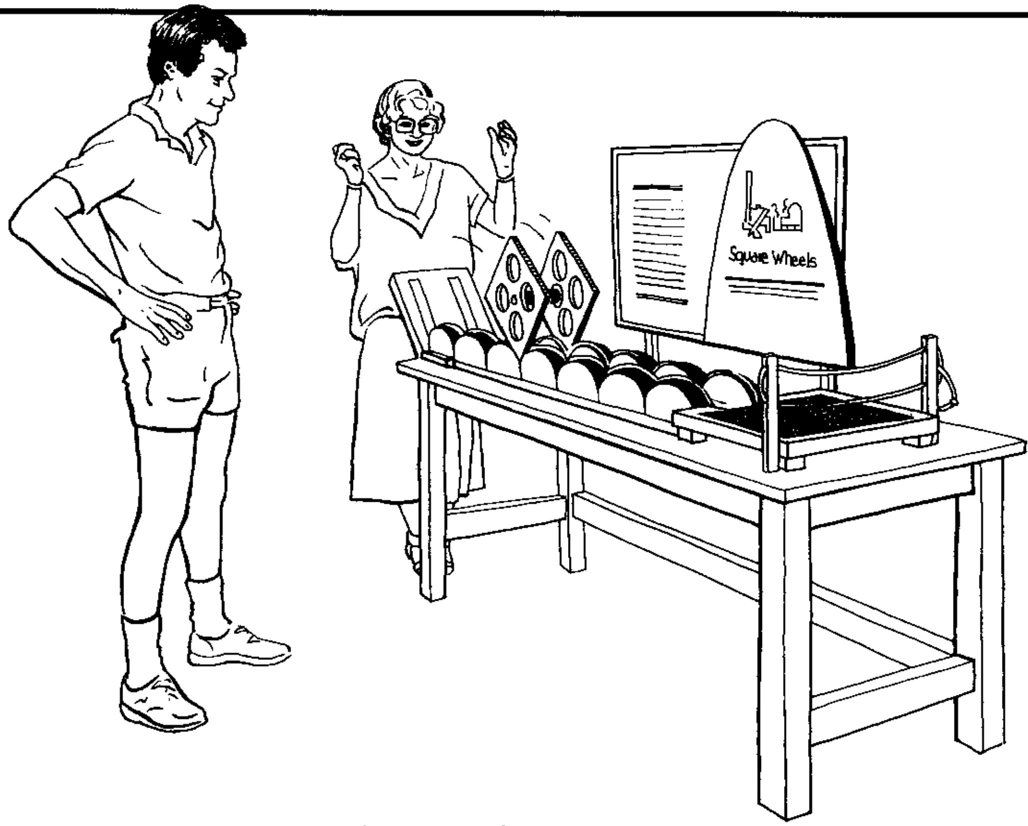
Exploratorium Cookbook III

A Construction Manual for Exploratorium Exhibits

by Ron Hipschman

Square Wheels



Description

Have you ever seen the cartoon of the caveman scratching his head and looking at a square wheel? Well, it would have worked if the caveman had had a specially shaped street on which to use it. This exhibit demonstrates that a square wheel will roll smoothly on a surface with properly spaced bumps of the right size and shape. The visitor starts a square wheel rolling from a slanted launching ramp (which aligns the wheel properly) over a scalloped road. The wheel is observed to roll smoothly with the axle at a constant height. The wheel will also balance wherever it is left, even if it is on the side of a bump (assuming it is aligned properly to start with).

Construction

The trick to this exhibit is getting the correct "bump" for your wheel. The bumps are catenary curves, and the exact catenary depends on the size of the square wheel that you choose. To plot out the catenary, use the following equation:

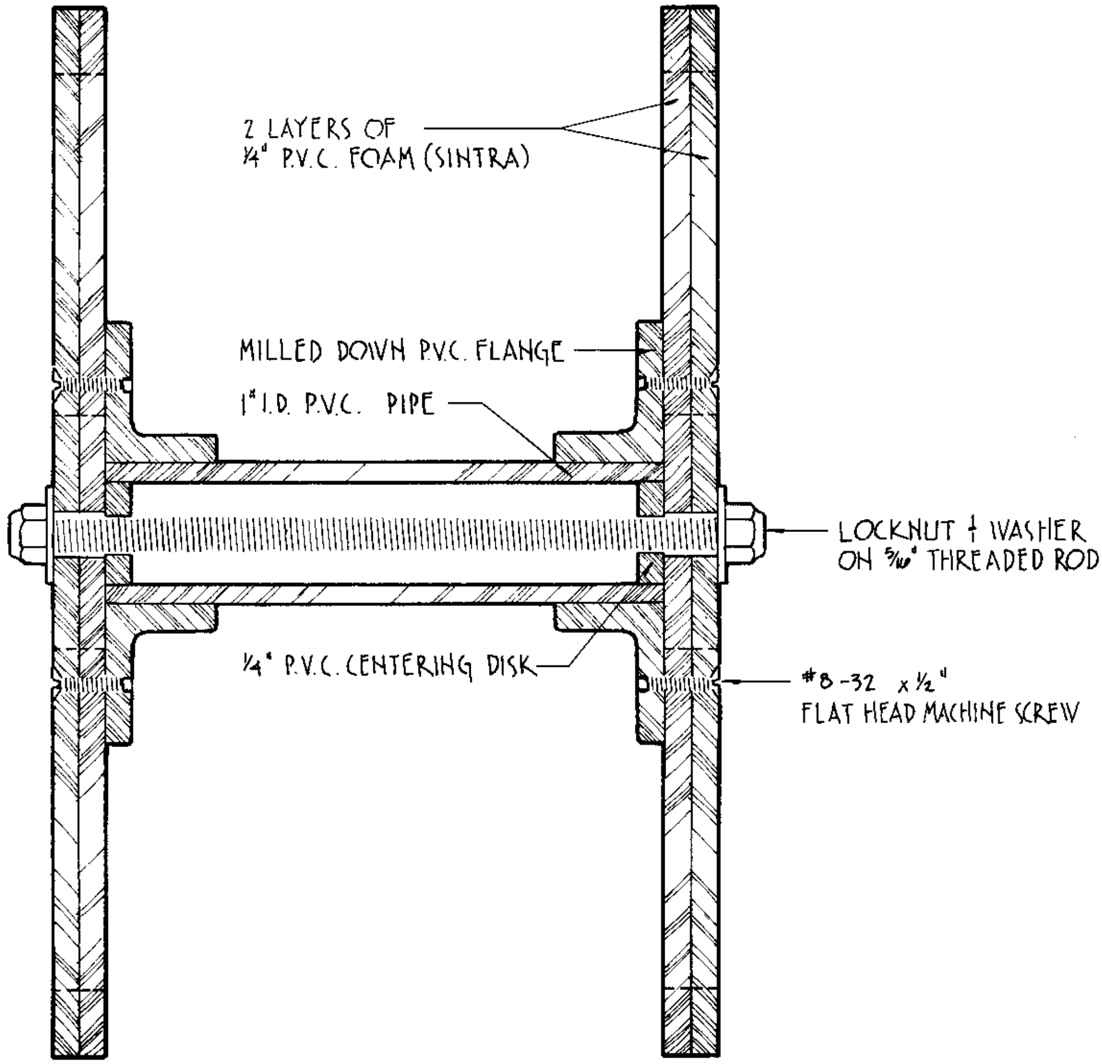
$$y = a \cosh(x/a) = a/2 \, (e^{(x/a)} + e^{(-x/a)})$$

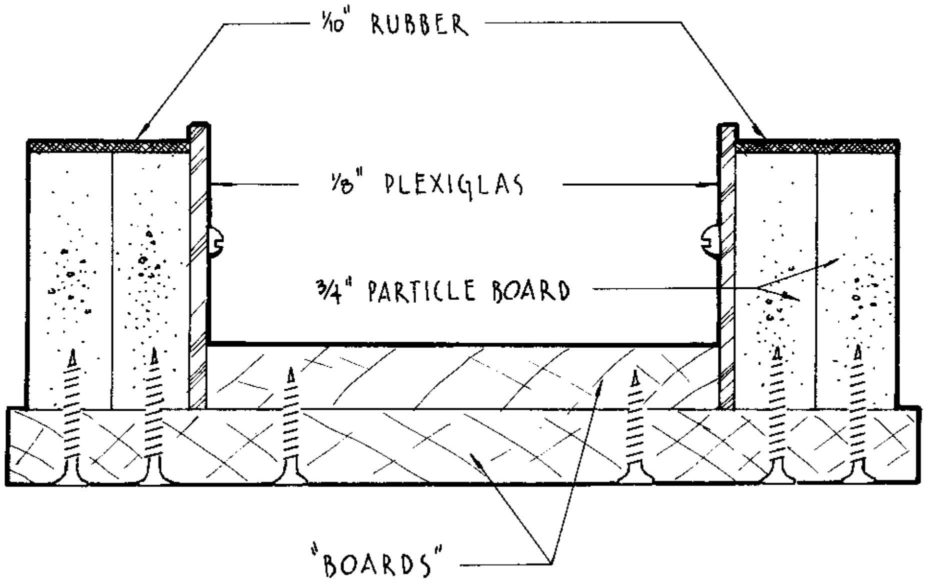
where "a" is half the length of the square wheel's side (or the distance from the center of any polygon to the the middle of one of its sides—see Critique and Speculation). To avoid any distortion of the curve, plot this out on a piece of 1/8" to 1/4" thick aluminum plate. This plate will become a router template for cutting the wooden bumps, after the "theoretical curve" described in the formula is adjusted to the "actual curve" necessary for the final rubber-covered bump (see below).

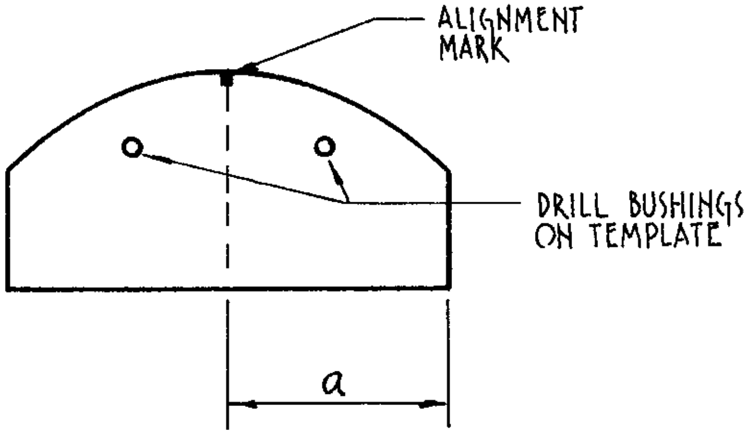
The horizontal dimension for each bump depends on the size of the square wheel, with the length along the catenary curve being equal to the length of the square wheel's side. To find the horizontal dimension of the bump, we differentiate the above equation and solve for where the slope equals 1. This is where the square wheel is balanced on its corner at the junction of the two bumps. The derivative of the cosh function is simply the sinh function.

Below is an example for our wheels, which are 10" across ($a = 5$):

Solve $Y' = \sinh(x/a)$ for x, with $Y' = 1$ (the slope) and $a = 5$


2 LAYERS OF
1/4" P.V.C. FOAM (SINTRA)
MILLED DOWN P.V.C. FLANGE
1" I.D. P.V.C. PIPE
LOCKNUT & WASHER
ON 5/16" THREADED ROD
1/4" P.V.C. CENTERING DISK
#8-32 x 1/2"
FLAT HEAD MACHINE SCREW


1/10" RUBBER
1/8" PLEXIGLAS
3/4" PARTICLE BOARD
"BOARDS"



You first solve for sinh(x/a) = 1 by using a calculator or computer that does sinh (and if you're lucky, arc-sinh)—or you could find the answer from a table in a book (and be snubbed by all of us computer hackers). This gives you x/a = 0.8814. Now solve for x by multiplying by a. The result, x = 4.407 inches, is the half-width of the bump.

Once you have this "theoretical" curve, you can begin to figure out the actual curve from which the real bumps will be cut. The difference between the theoretical and actual curves occurs because the wooden curves are made non-slip with a layer of rubber sheeting. The thickness of the rubber sheet must be taken into account before making the actual wooden bumps. To do this, take your theoretical curve and draw a parallel curve below it, with the distance between the two curves matching the thickness of the rubber sheet. Since we wrap the rubber sheet around each wooden bump, the same thickness must be subtracted from the bump's sides.

Be sure to sand the curve of the template smooth, as any irregularities will be duplicated when you route the wooden bumps. We drilled two holes in the aluminum and inserted steel drill bushings (see diagram), which make the template a drill guide as well as a router guide, and insure that all holes are drilled uniformly. Once the registration holes have been drilled in the wood, alignment bolts are inserted and the curve is cut. Use a ball bearing router bit on a router table. Note that only one 3/4" thick piece of particle board can be routed at a time (at least with our router). The registration holes can be used when putting the bumps together.

Our wooden bumps are 1-1/2" thick, made of two layers of 3/4" particle board screwed together. We use a total of 14 bumps with 7 on each side for a road about 5 feet in length. (This of course means that we had to route 28 3/4" pieces.) Apply the rubber strips to the wooden bumps with Scotch 90 spray adhesive. The bumps are screwed (from below) onto a "roadbed" board (see diagram). Another board between the two rows of bumps helps to stabilize the plexiglas guides and to keep everything vertical. The width of this board depends on the spacing between your wheels. The 1/8" plexiglas guides are sandwiched between the bumps and center board; they help keep the wheel on track.

To help the visitor start the wheel properly, we attached a 45 degree ramp flush to the left side of the bumps. Here the wheel can be laid flat and lined up with the first bump. A horizontal carpeted landing zone is also provided at the right end of the road.

The wheel assembly is a plastic construction. Four oversize square foamed-PVC plastic wheels are cut from 1/4" thick stock, glued together into 1/2" thick pairs, and then cut to their final 10" size. Large circles were cut out of the wheels to lighten them. The plastic used in the wheels, selected for its lightness, is called "Sintra" and is made in Europe by Alusuisse Metals, Inc.; we got ours from:

Pacific Coast Plastics, Inc.
1235 Howard St.
San Francisco, CA 94103
telephone: (800) 227-4011 (outside CA) (415) 864-2252 (in CA)

The two wheels are held together with a 1-1/4" diameter PVC pipe glued into PVC flanges (machined), which are in turn bolted to the square wheels. A long piece of threaded rod fastened with captive nuts passes through the center of the wheel (see diagram). The threaded rod keeps the wheels together and the four bolts that screw into the PVC flange keep the wheels from rotating relative to one another; center-drilled disks in the ends of the pipe help align the threaded rod. Be sure to carefully position the wheels on the axle so that the edges of both wheels are parallel. If the two wheels are rotated relative to one another, they won't roll smoothly. The corners of the wheels are rounded slightly (about 1/16" to 1/8" radius).

As an alignment aid, we put marks on the sides of the bumps at the tops of the curves and also on the midpoints of the wheelsides. The visitor can align the wheel to the bump and notice that if rolled a small distance, the wheel will remain in a given position on that bump.

Critique and Speculation

The development of Square Wheels illustrates a common conflict that can occur among exhibit designers. One side of the design team crusades for elegance and precision, while the other faction fights for a sturdy exhibit that will never wear out. The plastic-on-rubber combination that we arrived at to put our wheels on the road was actually a compromise between these two ideals. For instance, sandpaper-like stair tread material on the bump surfaces, combined with a thin rubber tread on the wheel, makes for a better grip, without any creep, than the present solution; but it also wears out quite fast and is too much of a maintenance problem. Solid plastics and metal are more durable than the foam plastic we use for the wheels, but don't have the advantage of lightness that you get with the Sintra.

The wheel surfaces become glazed periodically, requiring a light sanding to roughen them up. This of course makes the wheels slightly smaller each time, and eventually they have to be replaced. Since they also get bashed around some, you should be prepared to replace them at regular intervals and be set up to make new ones easily.

This idea of non-round wheels can be extended to any polygon except the triangle, whose corners are too sharp and get caught in the gullies. Obviously, the more sides the polygon has, the more it will resemble a circle and the smaller the road bumps will be—and the exhibit's dramatic impact will decrease proportionately.

Related Exploratorium Exhibits

Shape

Faces or Flower Vases; Peripheral Vision; Seen Clearly in Hazy Conditions; Whirling Watcher; Visible Magnetic Domains; Discernibility/Going to Pieces; Lumen Illusion; Triple-Aye Light Stick; Inferno; Recollections.

Composition of Circular Motion

Differential; Model Differential; Pacific Gas and Leather; Ring Toss; Rotating Light; Two Wheels and a Ball.

Moment of Inertia

Scaling; Downhill Race; Pendulum Table; Adjustable Plaything; Chaotic Pendulum; Drawing Board; Balancing Stick.

Square Wheels

This square wheel rolls smoothly over bumps shaped like catenary curves.

To do and notice

Place the square wheel on the far left end of the bumpy road. Push the wheel to the right and then let it go. Notice that the wheel rolls along smoothly and that the axle stays at a constant height.

Balance the wheel on top of one of the bumps and then nudge it very gently to one side or the other. Notice that the wheel rests in this new position rather than toppling.

What's going on

The shape of each bump in the road is a catenary curve, the same type of curve found in our *Catenary Arch* exhibit. As the square wheel rolls over these bumps, its center of gravity is always over the point where the square touches the bump. This means that the wheel is always balanced, so that its center of gravity remains at the same height. Since the wheel is always balanced, it keeps rolling with only a small push, just like a cylinder rolling along a flat surface. The wheel also tends to stay put when it is stopped. As long as the wheel is balanced (that is, as long as its center of gravity is supported) there is no reason for the wheel to move, even when it is perched on the side of a bump.

You can make the wheel seem to roll up a bump by shifting it to an unbalanced position. Place the wheel so that a corner is lodged in the crack between two bumps, and then slide it uphill about a quarter inch and let it go. The wheel will roll up over the bump you left it on, stopping only when its corner runs into the next bump.

So what

A great number of differently shaped wheels can be made to roll smoothly over an appropriately shaped road. All regular polygons will roll smoothly over a series of catenary bumps if the bumps have the right dimensions. In this case, another size square wouldn't work because the length of a path over the bump must be equal to the length of the square's sides. Triangular wheels won't work at all because the corners get caught in the valleys between the bumps.

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